UNIVERSITÉ DU LUXEMBOURG

LUXEMBOURG INSTITUTE OF HEALTH
RESEARCH DEDICATED TO LIFE
Multiomics Data Science

Testing Hypotheses about Means

from BSc course Biostatistics (UL)

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Hypotheses

Null and Alternative Hypotheses

When statisticians would like to make a claim, they do this in a form o hypothesis testing.

In hypothesis testing we begin by making a tentative assumption about a population parameter, i.e. by formulation of **a null hypothesis**.

Null hypothesis

The hypothesis tentatively assumed true in the hypothesis testing procedure, H_0

For safety reasons, we assume a situation when nothings "interesting" happens as H_0

Alternative hypothesis

The hypothesis concluded to be true if the null hypothesis is rejected, H_a

H_a will be a situation when we see something unusual, which require action

Hypotheses in a simplest case: comparing mean to a constant

One-tailed	One-tailed	Two-tailed
H_0: $\mu \le$ const	H_0: $\mu \ge$ const	H_0: $\mu =$ const
H_a: $\mu >$ const	H_a: $\mu <$ const	H_a: $\mu \ne$ const

Hypotheses

Type I Error

Type I error
The error of rejecting H_0 when it is true.

Type II error
The error of accepting H_0 when it is false.

Level of significance
The probability of making a Type I error when the null hypothesis is true as an equality, α

poor sensitivity

False Negative, β error

		Population Condition	
		H_0 True	H_a True
Conclusion	Accept H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

False Positive, α error

poor specificity

Hypotheses

One-tailed Test

One-tailed test

A hypothesis test in which rejection of the null hypothesis occurs for values of the test statistic in one tail of its sampling distribution

H_0: $\mu \le \mu_0$ $\qquad$ H_0: $\mu \ge \mu_0$

H_a: $\mu > \mu_0$ $\qquad$ H_a: $\mu < \mu_0$

A Trade Commission (TC) periodically conducts statistical studies designed to test the claims that manufacturers make about their products. For example, the label on a large can of Hilltop Coffee states that the can contains **3 pounds** of coffee. The TC knows that Hilltop's production process cannot place exactly 3 pounds of coffee in each can, even if the mean filling weight for the population of all cans filled is 3 pounds per can. However, as long as the population mean filling weight is at least 3 pounds per can, the rights of consumers will be protected. Thus, the TC interprets the label information on a large can of coffee as a claim by Hilltop that the population mean filling weight is at least 3 pounds per can. We will show how the TC can check Hilltop's claim by conducting a lower tail hypothesis test.

μ_0 = 3 lbm

Suppose sample of n=36 coffee cans is selected. From the previous studies it's known that σ = 0.18 lbm

Hypotheses

One-tailed Test: Example

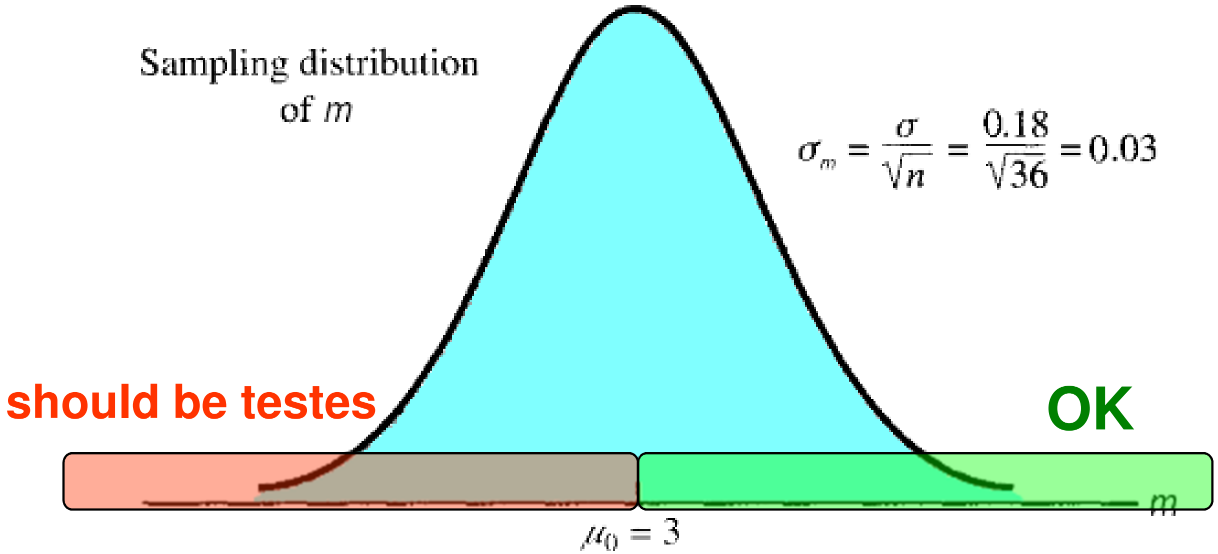
μ_0 = 3 lbm

Suppose sample of n = 36 coffee cans is selected and m = 2.92 is observed. From the previous studies it's known that σ = 0.18 lbm

H_0: $\mu \geq 3$ no action

H_a: $\mu < 3$ legal action

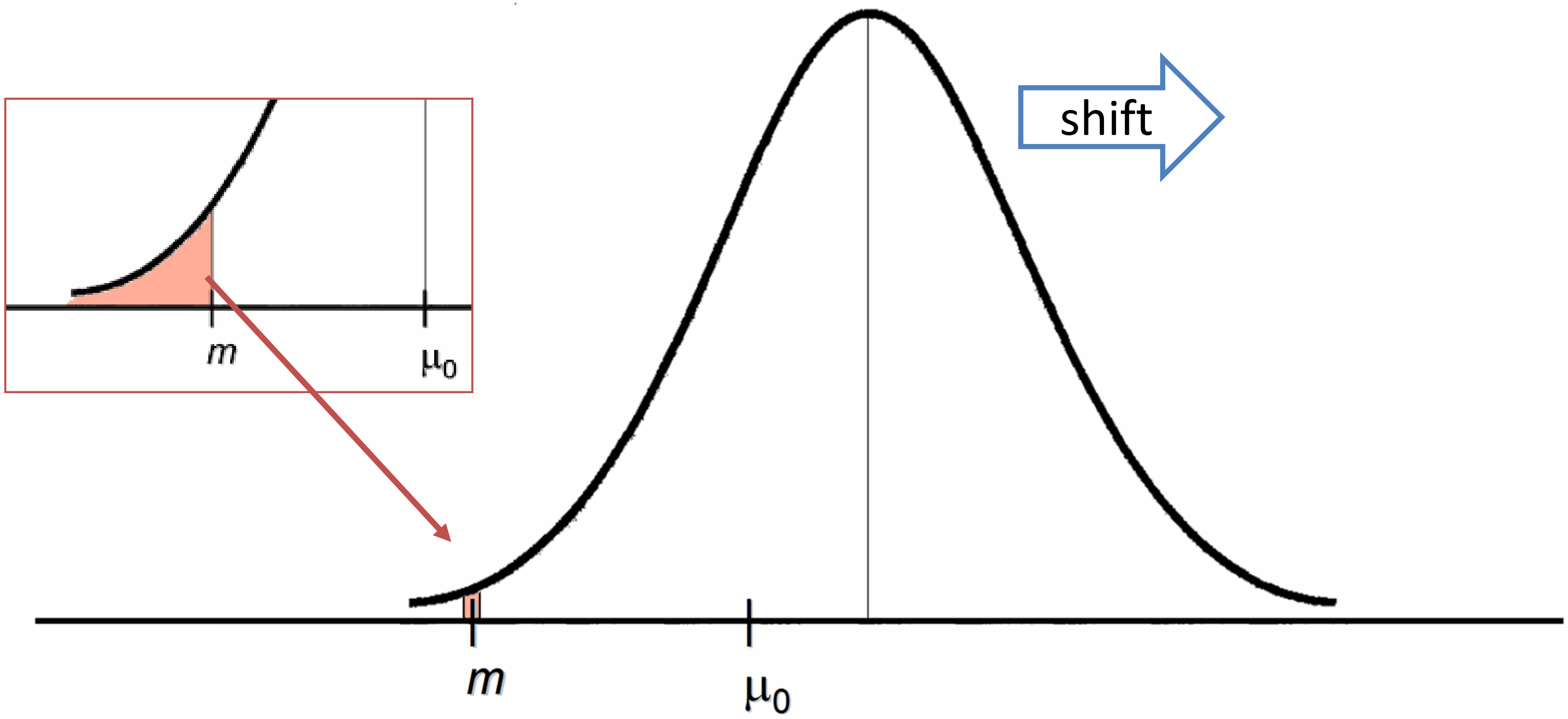
Let's say: in the extreme case, when μ=3, we would like to be 99% sure that we make no mistake, when starting legal actions against Hilltop Coffee. It means that selected significance level is $\alpha = 0.01$



P-value

Let's Try to Understand…

Let's find the probability of observation *m* for all possible $\mu \geq 3$. We start from an extreme case ($\mu=3$) and then probe all possible $\mu>3$. See the behavior of the small probability area around measured *m*. What you will get if you summarize its area for all possible $\mu \geq 3$?



P(*m*) for all possible $\mu \geq \mu_0$ is equal to *P*(*x*<*m*) for an extreme case of $\mu = \mu_0$

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P-value

Let’s Try to Understand…

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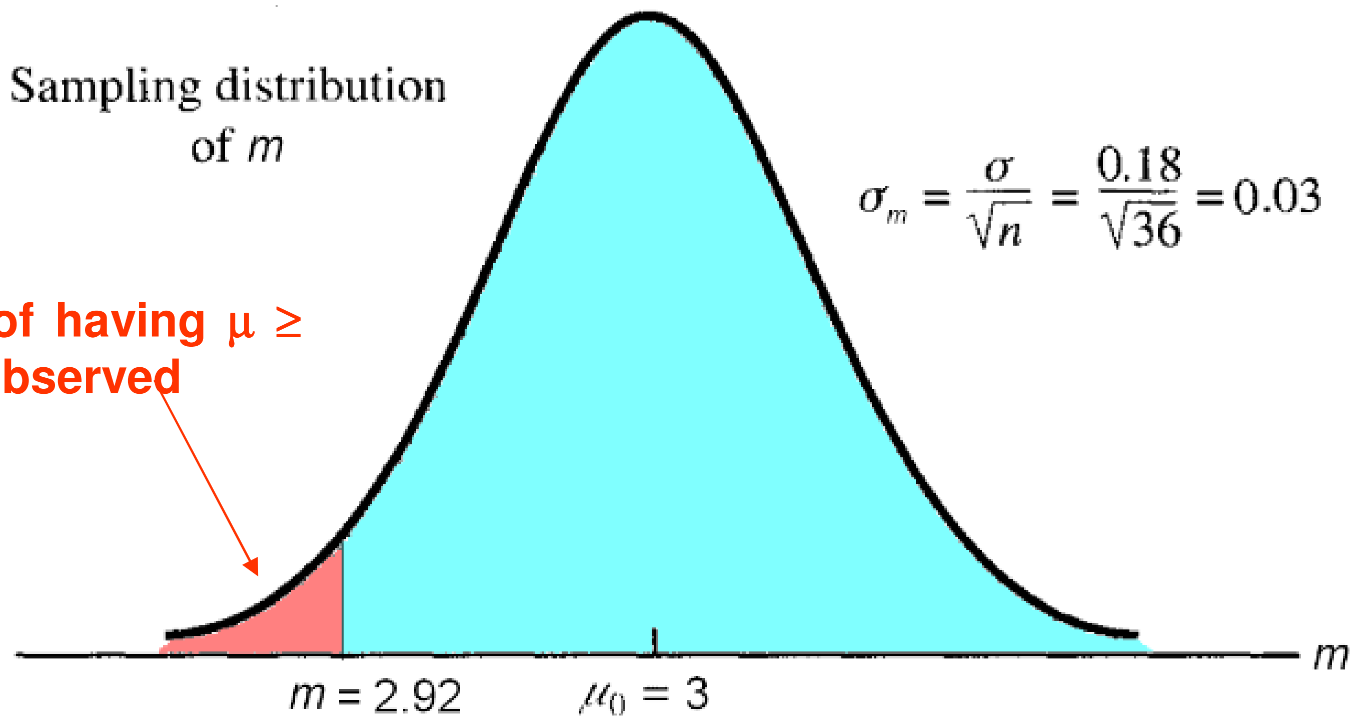
The probability of having μ ≥ 3, if *m* = 2.92 is observed



Red area characterizes the probability to observe, what we observed, if null hypothesis is true.

To be completely correct, the red area gives us a probability of making an error when rejecting the null hypothesis, or the p-value.

Hypotheses

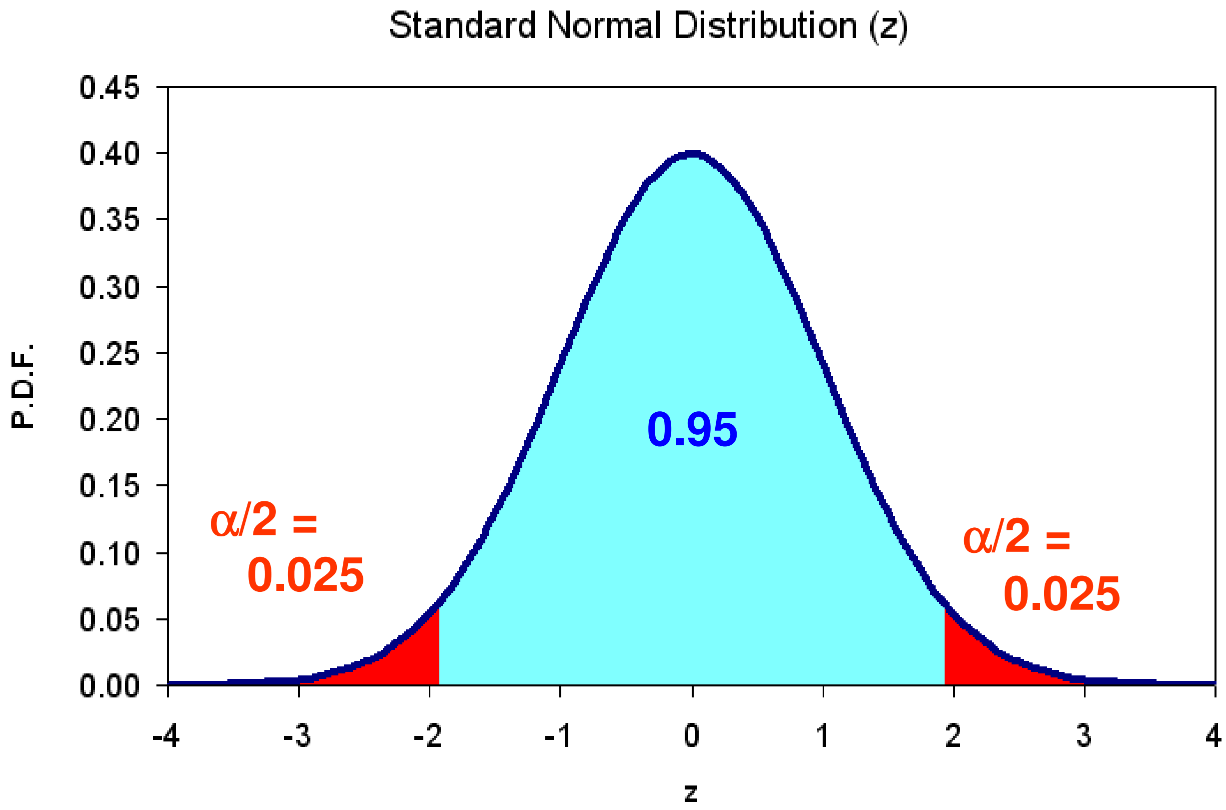
Two-tailed Test

Two-tailed test

A hypothesis test in which rejection of the null hypothesis occurs for values of the test statistic in either tail of its sampling distribution.

$H_0: \mu = \mu_0$

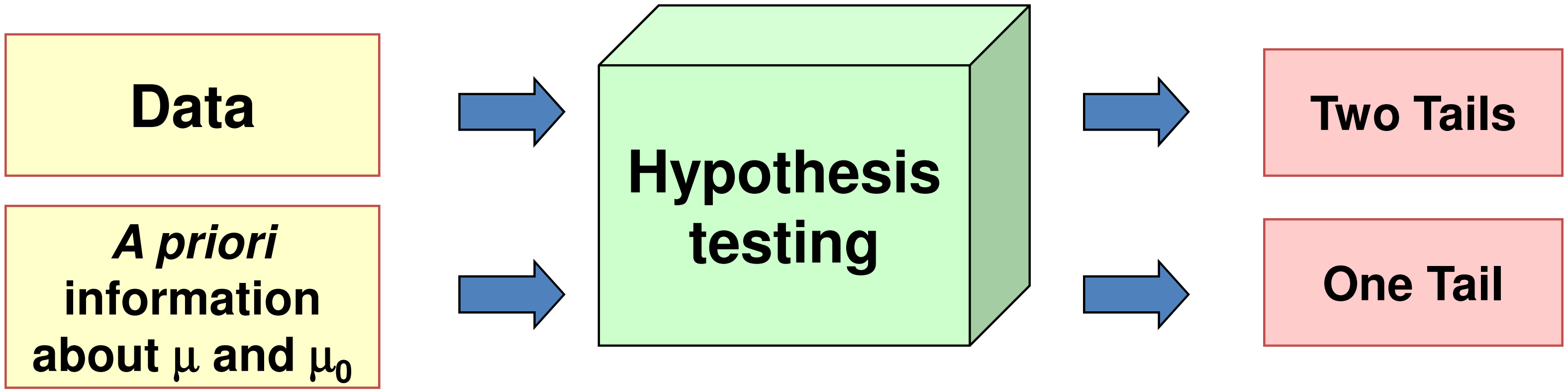
$H_a: \mu \neq \mu_0$



Hypotheses

One Tail Test vs. Two Tail Test

There is a raging controversy (for about the last hundred years) on whether or not it is ever appropriate to use a one-tailed test. The rationale is that if you already know the direction of the difference, why bother doing any statistical tests. While it is generally safest to use a two-tailed tests, there are situations where a one-tailed test seems more appropriate. The bottom line is that it is the choice of the researcher whether to use one-tailed or two-tailed research questions.



$$2 \times \text{p-value}_{(1\ \text{tail})} = \text{p-value}_{(2\ \text{tails})}$$

Two Populations

Independent Samples

Independent samples

Samples selected from two populations in such a way that the elements making up one sample are chosen independently of the elements making up the other sample.

Weight

Height

Smoking

simple T-test

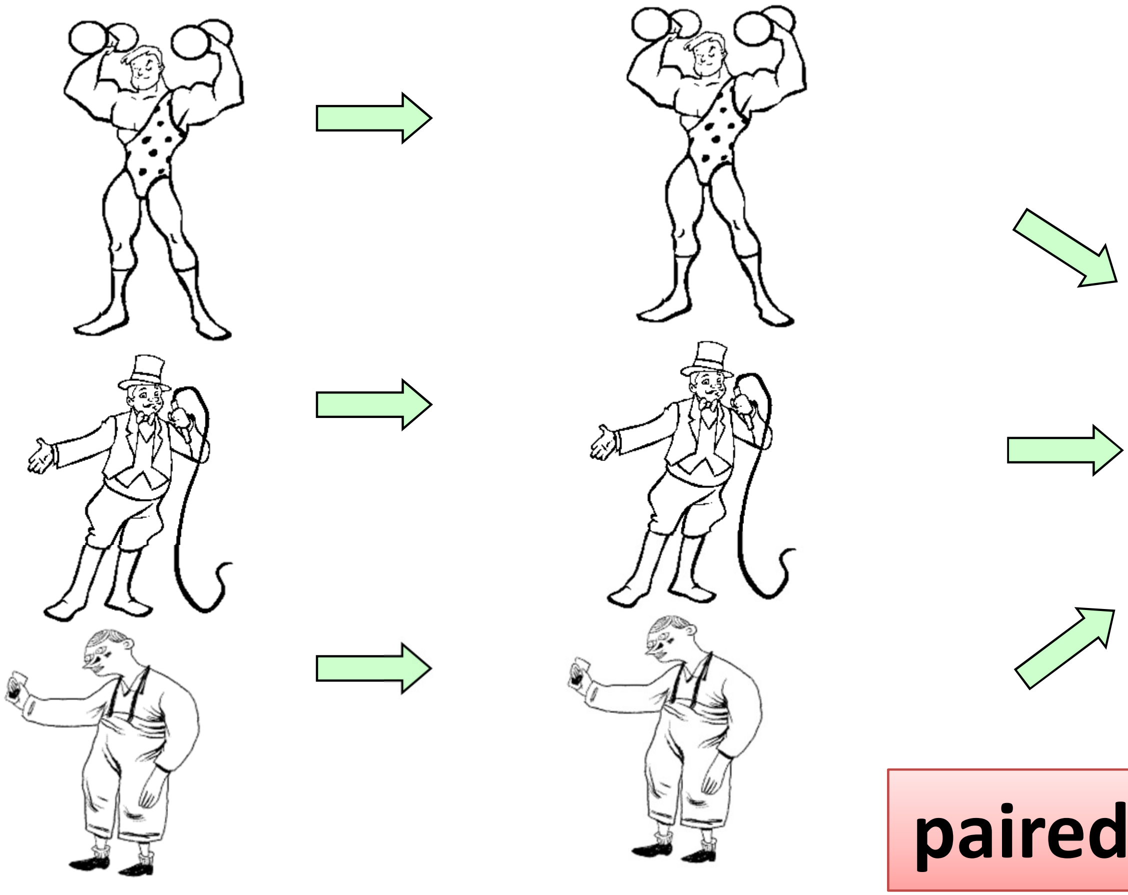
Two Populations

Dependent Samples

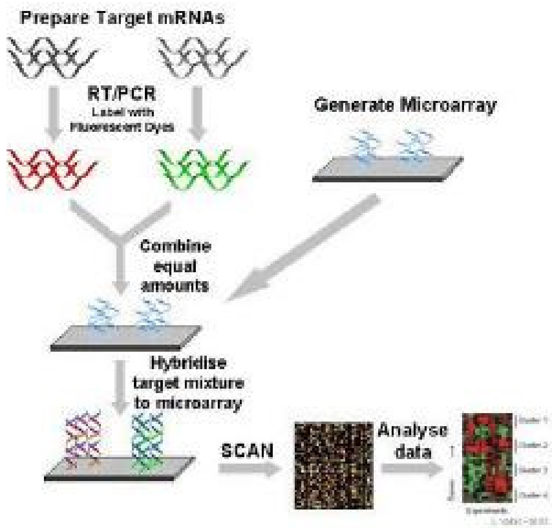
Matched samples

Samples in which each data value of one sample is matched with a corresponding data value of the other sample.

Before treatment **After treatment**



Analysis



paired T-test